Damage spreading in the mixed spin Ising model

Ce-Jun Liu* and H.-B. Schüttler†
Center for Simulational Physics, Department of Physics and Astronomy, The University of Georgia, Athens, Georgia 30602

Jia-Zhen Hu
Department of Applied Physics, Shanghai JiaoTong University, Shanghai 200030, People's Republic of China
(Received 6 July 2001; revised manuscript received 10 September 2001; published 17 December 2001)

We apply the damage spreading technique to study a mixed spin Ising model consisting of spin 1/2 and spin 1 with a crystal field interaction on the square lattice within a kind of Metropolis dynamics. The completely different behavior, depending on the value of the crystal field interaction, strongly suggests there may exist a dynamical tricritical point where the phase transition may change from the second order to the first order for this model.

DOI: 10.1103/PhysRevE.65.016114 PACS number(s): 05.50.+q, 64.60.−i, 75.40.Mg

I. INTRODUCTION

The damage spreading (DS) technique, i.e., measuring the Hamming distance between two different initial configurations as they evolve in time, was first studied in theoretical biology in the context of genetic evolution [1]. Later the DS concept found its way into the physics system [2–12]. In this method one essentially monitors the time evolution of two or more copies of the same system with different initial configurations subject to a specific dynamics and to the same thermal noise. It turns out that this method is less sensitive to the static fluctuations, when compared to the conventional Monte Carlo method where the time evolution of a single copy is investigated. The DS method has been successfully applied to many magnetic models, such as cellular automata, Ising ferromagnet, p-state clock, Potts, ANNNI, Ashkin-Teller, discrete N vector, XY, the Heisenberg, spin glasses, etc. Besides its wide variety of applications, the relationship between DS and the time-dependent thermodynamic properties in the Ising model [14], the possible connection of the damage transition (where the damage vanishes) and the percolation transition of geometrical clusters of correlated spins [15–17] are also investigated.

Most of the above systems which the damage spreading technique has been applied to are the systems with the second order phase transition. Also, the DS technique has been used to study the critical properties of the BEG model on a honeycomb lattice in the vicinity of tricritical line and how the DS technique may be applied to identify the tricritical point is also showed [18]. For the q-state Potts model on square lattice the DS studies also showed that it is possible to calculate the critical temperature of the model as well as to give some indication of the order of the phase transition [19]. The DS technique represents nowadays an important tool in the study of the dynamic as well as the static behavior in magnetic systems.

Here, we will use the DS technique to study a mixed spin Ising model (MSIM) on the square lattice in which two interpenetrating sublattices have spins one-half ($\pm 1/2$) and spins one ($\pm 1,0$). Mixed spin systems provide good results for studying ferrimagnetism. The magnetic properties of the mixed Ising models have been studied using high-temperature series expansions [20], renormalization group (RG) [21–23], mean field [24], effective field [25–27], Monte Carlo (MC) simulations and numerical transfer matrix calculations [28,29], and free-fermion approximation [30]. Besides at equilibrium conditions as stated as above, within mean field approach, the kinetics of the model in the presence of a time-dependent oscillating external field, has also been studied [31].

Among those works, there exist two opposite conclusions for the mixed spin Ising model on the square lattice: (i) the RG analysis [21], effective field theory [27], and the mean filed theory [24] indicate that there exists a compensation point or tricritical point at finite temperature; (ii) RG scheme [22] and MC and numerical transfer matrix technique [28,29] then got a contradictory conclusion. In this paper, the DS technique is applied to study the MSIM on the square lattice. We found that the mixed spin Ising model may exhibit a tricritical point at finite temperature.

In Sec. II we describe the model and the damage spreading technique that we will use. In Sec. III we present the results for the ''pure'' mixed spin Ising model in which the crystal field interaction is zero. On the basis of Sec. III, we perform the DS calculations for the MSIM with nonzero crystal interaction in Sec. IV. Finally, Sec. V presents the remarks and conclusions.

II. THE MODEL AND A KIND OF METROPOLIS DYNAMICS

We consider a mixed spin Ising model on the square lattice given by the Hamiltonian

$$H=-J\sum_{\langle ij\rangle}\sigma_i S_j+G\sum_j S_j^2, \qquad (1)$$

where the S takes the values ± 1 or 0 located in alternating sites with spins $\sigma=\pm 1$. The spins σ are spin 1/2, but we choose to put the factor of 1/2 into the interaction parameter.

*Email address: liu@hal.physast.uga.edu
†Email address: hbs@hal.physast.uga.edu

Each spin S has only σ as nearest neighbors and vice versa. The first summation is carried out only over nearest neighbor pairs of spins, and the second summation only runs over all sites of S sublattice. J is the exchange interaction, and G is the crystal field interaction parameter.

Our numerical simulations are implemented on a square lattice with N spins and linear size L ($N = L^2$ sites) subjected to periodic boundary conditions. A configuration of lattice spins at time t is

$$C(t)=\{\sigma(t),S(t)\}=\{\sigma_i(t),S_j(t)\},\quad i,j=1,2,3,\ldots,N/2. \tag{2}$$

In order to make a configuration $C(t)$ evolve in time, we use a kind of Metropolis dynamics that has already been applied to the $S=1/2$ and $S=1$ Ising model on a square lattice [32].

During each time interval $\delta t = 1/N$, one spin site i is chosen randomly. This site holds either a σ spin or an S spin. The spin value $\Delta_i(t+\delta t)$ at time $t+\delta t$ is then proposed by

$$\Delta_i(t+\delta t)=\begin{cases}1, & Z_{i1}(t)\leqslant 1/2\\ -1, & Z_{i1}(t)>1/2\end{cases} \tag{3}$$

if i is a σ spin site;

$$\Delta_i(t+\delta t)=\begin{cases}1, & 0\leqslant Z_{i1}(t)<1/3\\ 0, & 1/3\leqslant Z_{i1}(t)<2/3\\ -1, & 2/3\leqslant Z_{i1}(t)<1\end{cases} \tag{4}$$

if i is an S spin site. $Z_{i1}(t)$ is a uniform random number, $0 \leqslant Z_{i1}(t) \leqslant 1$.

One then updates the spin according to the following dynamics rule:

$$C_i(t+\delta t)=\begin{cases}\Delta_i(t+\delta t), & P_i(t)\geqslant Z_{i2}(t)\\ C_i(t), & \text{otherwise},\end{cases} \tag{5}$$

where

$$P_i(t)=\exp(-\Delta H_i/T) \tag{6}$$

$$\Delta H_i = H\{\Delta_i(t+\delta t)\}-H\{C_i(t)\}, \tag{7}$$

where $0 \leqslant Z_{i2}(t) \leqslant 1$ is another uniform random number, T is the temperature of the system in unit of J/K_B, G is in unit of J, and K_B is the Boltzmann constant.

We consider two different initial configurations $C^A(0)$ and $C^B(0)$ at time $t=0$, and let them evolve in time according to the above dynamics rule with the same sequence of random numbers for updating the spins. Then two configurations at time t, $C^A(t)$ and $C^B(t)$, are computed through the following Hamming distance (or damage) between them:

$$D(t)=\frac{1}{N}\sum_{i=1}^{N}\left[1-\delta(C_i^A(t),C_i^B(t))\right], \tag{8}$$

where $\delta(,)$ is the Kronecker delta function. Physically $D(t)$ measures the fractions of the spins that differ in the two replicas at time t. In calculations, we average $D(t)$ over many samples. The average distance is

$$\langle D(t)\rangle=\frac{1}{N_s}\sum_{j=1}^{N_s}D_j(t), \tag{9}$$

where $D_j(t)$ is the damage distance for the jth independent trial, N_s is the number of independent sample, the sum is over all trials [here, we have not used the conventional method, that is, the average was taken over only those samples whose $D(t)$ are not zero].

We also study the "damage susceptibility"

$$\sigma_{D(t)}=\sqrt{\langle D^2(t)\rangle-\langle D(t)\rangle^2}, \tag{10}$$

which measures the fluctuations of damage $D(t)$. We shall see that this quantity will provide a set of information to characterize different phases of the system, and it is very sensitive to the phase transition.

We will also investigate a quantity that we define as the ratio of $N_{S_j=0}$ (number of spins whose spin value $S_j=0$) to $N/2$ in S sublattice at the equilibrium state. We may regard it as the probability that one spin takes zero value in the S sublattice. In order to decrease the fluctuations, we take an average over those two replicas (configurations A and B)

$$\langle P_{S_j=0}\rangle=\frac{1}{N_s}\sum_{k=1}^{N_s}\left[\frac{N^A_{S_j=0}+N^B_{S_j=0}}{N}\right]_k, \tag{11}$$

where $\langle P_{S_j=0}\rangle$ depends on the temperature, time, initial conditions, and the noise. $\langle\,\rangle$ in Eq. (11) denote an average over many samples. It is actually another type of order parameter.

In the following calculations presented here, we take $J>0$ (ferromagnetic), the initial configuration is chosen to be

$$C^A(0)=-C^B(0)=1,\quad \forall\, i,j, \tag{12}$$

where we have assumed that there are no zero values in both S^A and S^B sublattices at $t=0$, i.e., $\langle D(0)\rangle=1$. Similarly, if we take $J<0$ (antiferromagnetic), i.e., the mixed Ising ferrimagnetic model, the initial condition could be chosen as: spins in S sublattice to be $+1$, spins in σ sublattice to be -1 in configuration A, spins in B then keep opposite to A. The calculations could have been done starting with other initial conditions, e.g., two random initial configurations. In this condition, the equilibrium of the system takes longer to be established at a low temperature, however, the results would be very similar [32].

We expect that these three quantities $\langle D(t)\rangle$, $\sigma_{D(t)}$, and $\langle P_{S_j=0}\rangle$, together with the temperature, initial condition, and any other parameters, will lead to the information about the criticality of the system.

III. RESULTS FOR MSIM WITH $G=0$

We first study the "pure" mixed spin Ising model with zero crystal field interaction. For this model, high-temperature series expansion [33], RG [21,23], the equiscale transformation [34], etc. have shown this model belongs to the same universality class as the standard $\sigma=1/2$ Ising ferromagnetic model. In the study of damage spreading method,

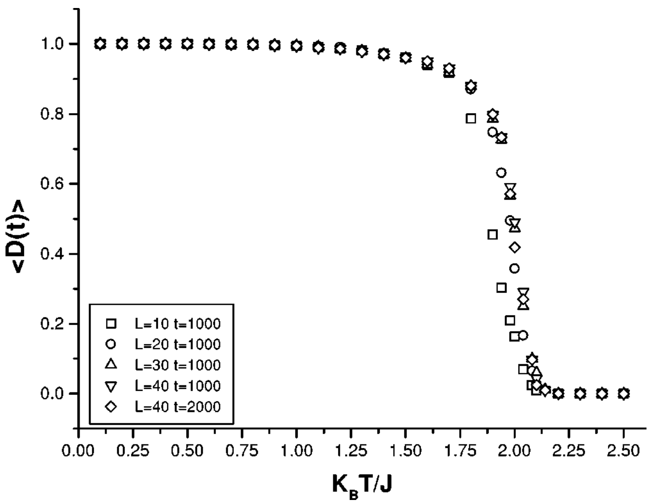

FIG. 1. Average damage $\langle D(t)\rangle$ as a function of temperature for the mixed spin Ising model with $G=0$ on the square lattice. The symbols for different times and the same lattice superimpose, indicating the establishment of the equilibrium.

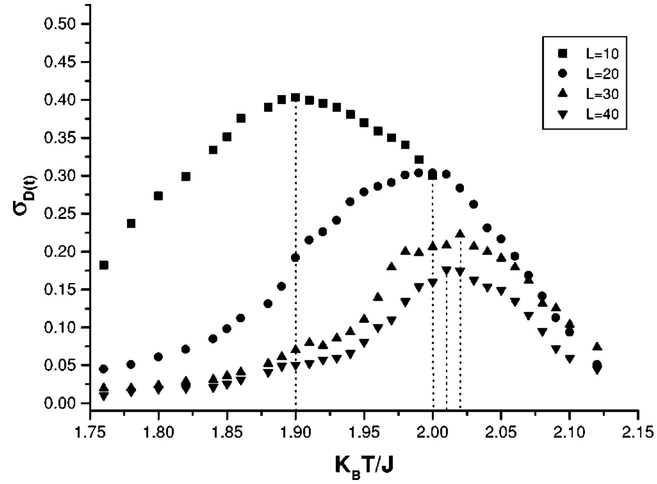

FIG. 2. Damage susceptibility $\sigma_{D(t)}$ as a function of temperature for mixed spin Ising model with $G=0$ on the square lattice at $t=1000$. The dotted lines indicate the positions of $T_\sigma(L)$. We estimate the damage spreading temperature T_σ from $\sigma_{D(t)}$ to be 2.02.

it is now generally believed that for the standard $\sigma=1/2$ Ising model, the dynamical phase (damage spreading) transition occurs at the same temperature as its corresponding static model.

In our DS procedure, the simulations have been performed for four lattice sizes. The results are averaged over 1500, 800, 400, and 200 samples for $L=10,\ 20,\ 30$, and 40, respectively. Figure 1 shows the $\langle D(t)\rangle$ as a function of the temperature and the time. We know that the equilibrium is reached at all temperatures except in the critical region where finite-size and finite-time effects can be seen. The shape of $\langle D(t)\rangle$ in Fig. 1 is very similar to the standard Ising model studied using DS technique within the heat bath dynamics and the same initial condition [11]. For the standard Ising model, within the heat bath dynamics and the initial condition (12), it has been proved that the damage distance $\langle D(t)\rangle$ is equivalent to the order parameter, the average magnetization, of the system at time t [13].

We clearly observe two distinct regions in Fig. 1: (i) a low-temperature region $(T<T_D, T_D\cong 2.0)$, where $\langle D(t)\rangle$ does not vanish for all cases; (ii) a high-temperature region $(T\geqslant T_D)$, where the $\langle D(t)\rangle$ vanish for all system sizes and time t. Two distinct temperature regions divided by a damage spreading transition temperature T_D are believed to denote the corresponding static continuous phase transition. The damage spreading features in Fig. 1 have been observed in most of the Ising-like systems.

We observed that the finite-time effect for $t=1000$ and $t=2000$ is relatively small for our chosen initial condition. In the following calculations, we will assume that the systems have reached their equilibrium states at $t=1000$ for the chosen initial condition (12).

In view of the temperature dependence of the fluctuation $\sigma_{D(t)}$, our simulation shows that there is an almost null fluctuation in the low- and high-temperature region, except near the damage spreading transition temperature T_D where it rises abruptly. Figure 2 shows the $\sigma_{D(t)}$ as a function of temperature for the four system sizes.

From the maximum values of $\sigma_{D(t)}$, we may locate the phase transition temperature T_σ. We estimate (run for several different random number sequences) $T_\sigma(L)$ to be 1.90 ± 0.05, 1.99 ± 0.04, 2.02 ± 0.05, and 2.02 ± 0.03 for $L=10,\ 20,\ 30$, and 40, respectively. From these data, we get a more accurate estimate for the damage spreading (dynamical phase transition) temperature than T_D, which is $T_\sigma \cong 2.02$ for this model.

The feature of the simulation result of $\langle P_{S_j=0}\rangle - T$ relationship for the model is that it increases as the temperature increases. As we have known from Eq. (1), the $\langle P_{S_j=0}\rangle$ may roughly reflect the relationship between internal energy U and the temperature T when $G=0$. Indeed, the S-like shape of $\langle P_{S_j=0}\rangle$ is very similar to the exact $U-T$ relationship for the standard Ising model on the square lattice.

In order to obtain a more reliable estimate for this dynamical transition temperature, we use the finite-size scaling procedure [12]. For each sample s, we calculate the distance $D_s(t)$ at times $t=1,2,3\ \ldots$. The calculation can always be stopped when the distance vanishes since it remains zero at any later time.

With definition one measures the characteristic time τ_1 and characteristic square time τ_2 as

$$\tau_1(L,T,s)=\sum_t tD_s(t)/\sum_t D_s(t) \tag{13}$$

$$\tau_2(L,T,s)=\sum_t t^2D_s(t)/\sum_t D_s(t) \tag{14}$$

and scaling form

$$\tau_1(L,T,s)\sim u(L)f_1(v(L)(T-T_c),s) \tag{15}$$

$$\tau_2(L,T,s)\sim u^2(L)f_2(v(L)(T-T_c),s). \tag{16}$$

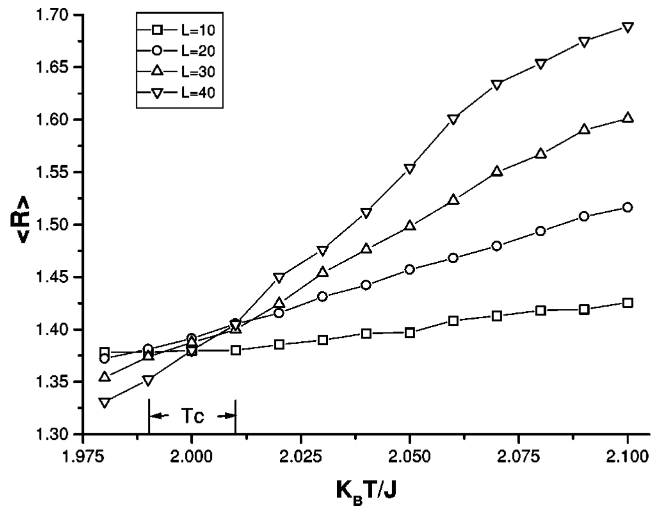

FIG. 3. Ratio $\langle R\rangle$ versus temperature T for different sizes L for the mixed spin Ising model with $G=0$ on the square lattice. The full line is a guide for the eye. All curves cross in the region of $T_c=2.000\pm0.010$.

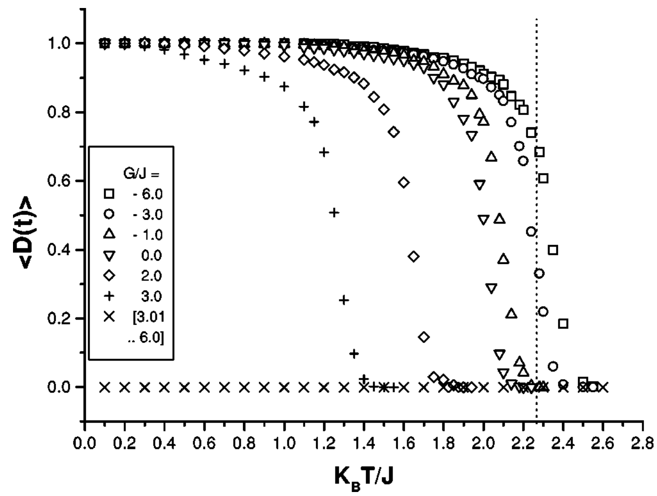

FIG. 4. Average damage $\langle D(t)\rangle$ as a function of temperature T and G for mixed spin Ising model with $G\neq 0$ on the square lattice at $L=40$, $t=1000$, and $N_s=200$. A completely different behavior of $\langle D(t)\rangle$ can be observed for $G/J\leqslant 3.0$ and for $G/J>3.0$. The vertical dotted line marks the exactly known value of T_c for the standard Ising model on the square lattice.

We can find that the ratio

$$\langle R(L,T,s)\rangle=\tau_2(L,T,s)/\tau_1^2(L,T,s)\sim f_3(v(L)(T-T_c)) \tag{17}$$

is independent of lattice size L at the dynamical transition temperature T_c when we take an average over many samples. This means that for large L, all the curves $\langle R\rangle$ plotted as function of T for different L should cross at the same temperature T_c, i.e., the dynamical critical temperature. In Fig. 3 we plot $\langle R\rangle$ averaged over 1500, 800, 400, and 200 samples against T for $L=10$, 20, 30, and 40, respectively. Our estimate of T_c for this mixed spin Ising model with $G=0$ and the initial condition (12) is $T_c\cong 2.000\pm0.010$. This result is consistent with $T_\sigma=2.02$ from $\sigma_{D(t)}$ and is also consistent with its static transition temperature at 1.9569 [30], and 2.016 ± 0.078 [33], etc.

We may use this method to estimate the dynamical critical exponent z defined at the dynamical critical temperature T_c as $\tau\sim L^z$, where L is the linear size and τ is the relaxation time for the dynamics [13]. Near the critical temperature, the fluctuation is very strong, and unlike the standard Ising model, we have no exact solution of T_c for this model. We expect rather large error bars in the estimate of the critical exponent z. We use τ_1 in Eq. (13) to measure the average vanishing time [7] at T_c and repeat the simulations on different sizes $L=10$, 20, 30, and 40. z is the slope of the $\ln(\langle\tau_1\rangle)$ versus $\ln(L)$. We have estimated z with the initial condition (12) for this "pure" MSIM to be

$$z=2.65\pm0.26 \quad (T_c=2.000). \tag{18}$$

So far, we have had the knowledge of "pure" mixed spin Ising model with zero crystal field interaction using DS technique. In the following section, we use the same DS method to investigate the MSIM when crystal field interaction is not zero. We find that the behavior of the model strongly depends on the crystal field interaction.

IV. RESULTS FOR MSIM WITH $G\neq 0$

We choose several values of G for the study using DS procedure. In the following calculations, we take the parameters to be $L=40$, $t=1000$, and $N_s=200$. Figure 4 shows the results of $\langle D(t)\rangle$ as a function of T and G.

We may observe a completely different behavior of $\langle D(t)\rangle$: for $G/J\leqslant 3.0$, the behavior of $\langle D(t)\rangle$ are very similar to the previous $G=0$ case (the $G=0$ result is also presented in the figure), there exist Ising-like continuous phase transition for the model for those G values; for $G/J>3.0$, contrary to the $G/J\leqslant 3.0$ cases, $\langle D(t)\rangle$ is zero for all temperature regions. From Fig. 4 we may estimate the approximate (continuous) dynamical transition temperature T_D to be 2.55, 2.5, 2.25, 2.2, 1.9, and 1.45 for $G/J=-6.0$, -3.0, -1.0, 0.0, 2.0, and 3.0, respectively.

In Fig. 5, we plot the $\sigma_{D(t)}$ curves. Similar things as in Fig. 4 can be observed: for $G/J\leqslant 3.0$, peaklike curves of $\sigma_{D(t)}$ can be seen, which are the features of second order transition; for $G/J>3.0$, $\sigma_{D(t)}=0$ for all temperature regions. We may get more accurate (continuous) dynamical transition temperatures than T_D to be $T_\sigma=2.3\pm0.035$, 2.25 ± 0.04, 2.19 ± 0.035, 2.02 ± 0.03, 1.62 ± 0.04, and 1.26 ± 0.02 for $G/J=-6.0$, -3.0, -2.0, 0.0, 2.0, and 3.0, respectively. Similarly, we may use the same finite-size scaling procedure as in Sec. III to get the improved dynamical transition temperature results.

The interesting features of $\langle P_{S_j=0}\rangle$ as a function of T and G/J are plotted in Fig. 6. We can also see that there exist two different regions for G values.

(i) $G/J\leqslant 3.0$, the $\langle P_{S_j=0}\rangle$ curves are very similar to the $G=0$ S-like curve. There exist continuous phase transitions within those G values. In the limit of $G\to-\infty$, this model is reduced to the standard Ising model. S and σ can only take $+1$ or -1 values.

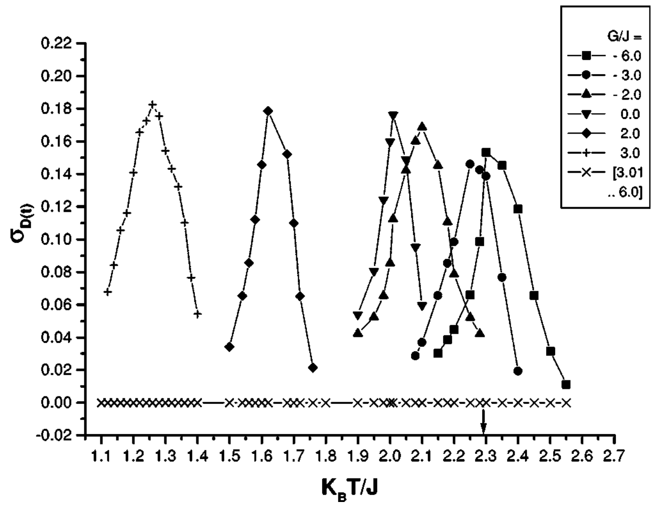

FIG. 5. Damage susceptibility $\sigma_{D(t)}$ as a function of temperature and G for mixed spin Ising model with $G \neq 0$ on the square lattice at $L=40$, $t=1000$, and $N_s=200$. A completely different behavior can be observed for $G/J \leqslant 3.0$ and for $G/J > 3.0$. The full line is a guide to the eye. The arrow points to the transition temperature of standard Ising model on the square lattice.

(ii) $G/J \geqslant 3.01$, contrary to $G/J \leqslant 3.0$ cases, $\langle P_{S_j=0} \rangle$ decreases as the temperature increases. In the limit of $G \to +\infty$, this model reaches to a new phase that we may call the staggered quadrupolar phase, similar to a phase in the BEG model [35]. In this phase, S sublattice has $S_j=0$ at every site and the σ sublattice has sites occupied randomly by $\sigma_i = +1$ or -1. This phase can be reached at low temperature $T=0.1$ and $G/J=5.0$ as shown in Fig. 6 in our DS dynamics approach.

We may explain the behavior of this model as follows. In this model, we only consider the nearest-neighbor interaction between S and σ spins. When G becomes large, the probability for S sublattice spins to take the zero values increases, the interaction of the total system becomes weak, and the second order phase transition temperature also becomes small. However, when the interaction of the system is too small, it cannot support the long range order of the system, i.e., there is no continuous phase transition. We may analyze this problem from another angle. If we regard the $S_j=0$ state as a "hole," then the S sublattices are occupied by $\sigma = \pm 1$ and the holes. Parameter G can change the relative number of σ spins and the holes in the S sublattices (it has the meaning of chemical potential). When $G \to -\infty$, there is no hole in the S sublattice. Both sublattices are occupied by σ spins, corresponding to the standard Ising model. When G increases, the number of holes increases. When $G \to +\infty$, the S sublattices are all

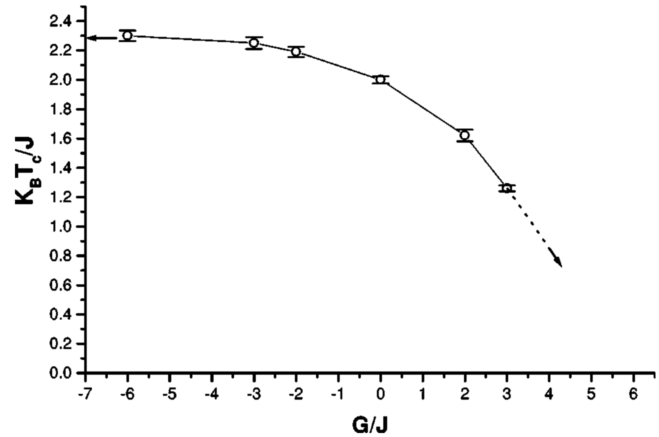

FIG. 7. Finite-temperature phase diagram for the mixed spin Ising model on the square lattice using damage spreading procedure. The solid line corresponds to the second order transition. The dotted line represents the first order transition (and only serves as a guideline). The upper-left arrow points to the transition temperature of the standard Ising model.

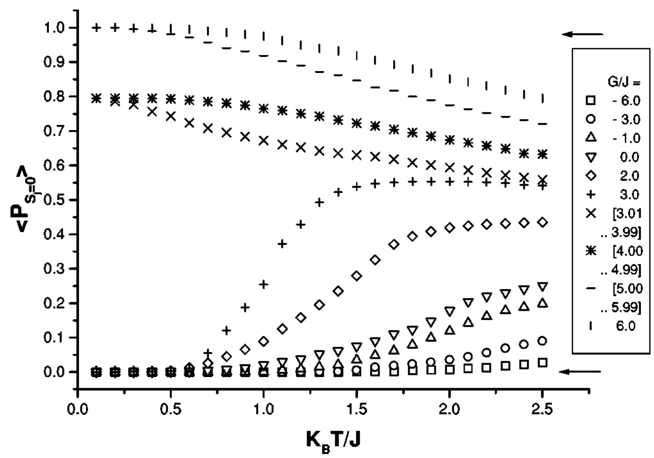

FIG. 6. Ratio $\langle P_{S_j=0} \rangle$ as a function of T and G for the mixed spin Ising model on the square lattice at $L=40$, $t=1000$, and $N_s=200$. Two completely different regions can be observed for $G/J \leqslant 3.0$ and for $G/J > 3.0$. The upper arrow points to the $\langle P_{S_j=0} \rangle = 1.0$ that corresponds to the staggered quadrupolar phase. The lower arrow points to the $\langle P_{S_j=0} \rangle = 0.0$ that corresponds to the standard Ising model.

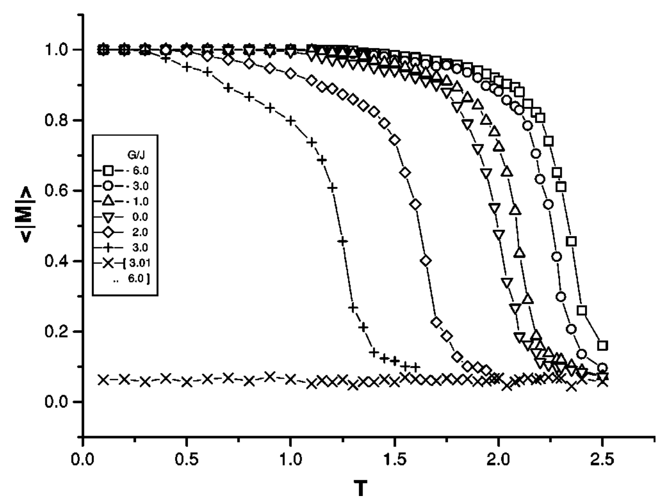

FIG. 8. Total magnetization of the system $\langle |M(t)| \rangle$ as a function of T and G for the mixed spin Ising model on the square lattice at $L=40$, $t=1000$, and $N_s=200$. Similar to $\langle D(t) \rangle$, two completely different regions can be observed for $G/J \leqslant 3.0$ and for $G/J > 3.0$. For each $G/J > 3.0$ value, $\langle |M(t)| \rangle$ independently goes to zero, except for the remanent finite-size and finite-time effects.

occupied by $S_j=0$ spin states. At the equilibrium state, for the G changes between $-\infty$ and $+\infty$, the holes can move and they have a tendency to gather to form clusters in order to make the system stable. When the G increases to a critical value, σ spins can no longer form the infinite clusters, then there is no long range order and no continuous phase transition. However, the finite σ spin clusters and the formation of the equivalent "hole clusters" can support the first order phase transition for the system. Here, the so-called "hole cluster" means that the spins of the S sublattice within a hole cluster are all occupied by hole ($S_j=0$). Although the σ spins occupy the σ sublattice within a hole cluster—they have no interactions among them—it is equivalent to the cluster which is completely composed of holes. The configuration of spins can verify this point.

According to our DS results, we may schematically plot the finite-temperature phase diagram for this model, and use T_σ obtained from $\sigma_{D(t)}-T$ relationship in Fig. 5 as the (second order) phase transition temperature. In Fig. 7, the general shape of the phase diagram shows reasonable agreement between our results and the Monte Carlo simulations [28], except for the range of $G/J>3.01$ where the first order transition could occur in our DS approach. From the data of our calculations, we estimate the tricritical point to be $(T_{tri},\ G_{tri}/J)=(1.26\pm 0.02,3.00)$. The known static values for the tricritical point for this mixed spin Ising model on the square lattice are (1.232, 4.198) [21] and (0.9936, 3.9376) [27], etc.

We have also performed the same calculations for the $J<0$ mixed Ising ferrimagnetic model (using the initial condition stated in Sec. II such that $\langle D(0)\rangle$ is still 1), and the very similar features have been obtained. Therefore, we expect that there exist both the second and the first order phase transitions for this mixed spin Ising model on the square lattice depending on the values of G.

V. REMARKS AND CONCLUSION

The mixed Ising ferrimagnetic system is relevant for understanding bimetallic molecular ferrimagnets that are synthesized in search of stable, crystalline materials, with spontaneous magnetic moments at room temperature [36]. In this paper, we investigate, through the damage spreading technique, the dynamical behavior of the mixed spin Ising model on the square lattice within a kind of Metropolis dynamics. We find that the behavior of the system strongly depends on the values of the crystal field parameter G. For a large and negative value of G, the systems behave similar to the standard Ising model and have continuous phase transitions. For a large and positive value of G, the model reaches to a staggered quadrupolar phase where the S sublattice sites are completely occupied by $S_j=0$. When G changes between $-\infty$ and $+\infty$, our damage spreading result strongly suggests the existence of a tricritical point.

We should point out that, unlike the conventional criteria for the first order transition, the damage spreading results here seem not to give us the explicit evidence for the first order transition, such as, an obvious discontinuity for the order parameter. In Fig. 8, we calculate one thermodynamic order parameter, the total magnetization of the system using the same initial condition of Eq. (12) (we take an average over A and B configurations). Its behavior is quite similar to the $\langle D(t)\rangle-T$ relationship. We do not observe the obvious discontinuity or "jump" for $\langle|M(t)|\rangle$, $\langle D(t)\rangle$, or $\langle P_{S_j=0}\rangle$ at the tricritical point $G/J=3.0$ where we regard it as the meeting point of the second order and the first order transition lines, even if we may claim that at $G/J=3.0$ those three quantities have had the largest "jump" in the critical region when compared to other $G/J<3.0$ cases as seen in Figs. 4, 6, and 8.

Very recently, we used the same DS dynamics to calculate the $S=1$ Blume-Capel model on the square lattice where only one spin variable S_i takes 0, or ± 1 for each lattice site with the same form of the Hamiltonian as in Eq. (1). The $S=1$ Blume-Capel model has well-known tricritical behavior as shown in Ref. [37]. From our simulations of this model (not shown in this paper), we got quite the same shapes and features for these quantities, $\langle D(t)\rangle$, $\sigma_{D(t)}$, $\langle P_{S_j=0}\rangle$, and $\langle|M(t)|\rangle$ as the mixed spin Ising model, except at the tricritical point (i.e., above meeting point). An obvious discontinuity or "jump" of $\langle D(t)\rangle$, $\langle P_{S_j=0}\rangle$, and $\langle|M(t)|\rangle$ can be observed for the Blume-Capel model. Also, the estimated dynamical tricritical point for this Blume-Capel model by DS technique is in excellent agreement with other approaches. Unlike the $S=1$ Blume-Capel model, we cannot observe an obvious discontinuity at the tricritical point in the mixed spin Ising model because there is only one $S=1$ sublattice for this model. Due to the small discontinuity in the mixed spin Ising model, we may also name this first order transition the "weak" first order transition (for a similar case the reader may refer to the q-state Potts model [38]), and we also expect that the results could be improved if system size L is increased. Further studies on this simple but fruitful model, the behavior of which is not yet well established, would be welcome.

ACKNOWLEDGMENTS

C.J.L. wishes to thank J. A. Plascak, Shan-Ho Tsai, D. P. Landau, S. P. Lewis for helpful conversations, and the Center for Simulational Physics, the Department of Physics and Astronomy, University of Georgia for the use of the computer facilities. This work was supported by U.S. National Science Foundation under NSF Grant No. DMR-9970291.

[1] S. A. Kauffman, J. Theor. Biol. **22**, 437 (1969).
[2] N. Jan and L. de Arcangelis, Annu. Rev. Comput. Phys. **1**, 1 (1994), and references therein.
[3] H. J. Herrmann, in *Computer Simulations in Condensed Matter Physics II*, edited by D. P. Landau, K. K. Mon, and H. B. Schüttler (Springer, Berlin, 1989), Vol. 45, p. 56.
[4] F. G. B. Moreira, A. J. F. de Souza, and A. M. Mariz, Phys. Rev. E **53**, 332 (1996).

[5] A. A. Júnior and F. D. Nobre, Physica A **243**, 58 (1997).
[6] P. Grassberger, J. Phys. A **547**, 214 (1995); U. Gropengiesser, Physica A **215**, 308 (1995); T. Vojta and M. Schreiber, Phys. Rev. E **58**, 7998 (1998).
[7] F. Wang, N. Natano, and M. Suzuki, J. Phys. A **28**, 4543 (1995).
[8] E. S. de Sousa, A. M. Mariz, F. D. Nobre, and U. M. S. Costa, Physica A **241**, 469 (1997).
[9] L. da Silva, F. A. Tamarit, and A. C. N. de Magalhães, J. Phys. A **30**, 2329 (1997).
[10] J. A. Redinz, F. A. Tamarit, and A. C. N. de Magalhães, Physica A **255**, 439 (1998).
[11] M. N. Barber and B. Derrida, J. Stat. Phys. **51**, 877 (1988).
[12] O. Golinelli and B. Derrida, J. Phys. A **22**, L939 (1989).
[13] A. U. Neumann and B. Derrida, J. Phys. (France) **49**, 1647 (1988).
[14] C. Glotzer, P. H. Poole, and N. Jan, J. Stat. Phys. **68**, 895 (1992).
[15] G. M. Zhang and C. Z. Yang, Europhys. Lett. **22**, 505 (1993).
[16] L. de Arcangelis, A. Coniglio, and F. Peruggi, Europhys. Lett. **14**, 515 (1991).
[17] M. Nicodemi, Physica A **238**, 9 (1997).
[18] U. M. S. Costa and I. V. Rojdestvenski, Phys. Lett. A **231**, 128 (1997).
[19] M. F. de A. Bibiano, F. G. Brady Moreira, and A. M. Mariz, Phys. Rev. E **55**, 1448 (1997).
[20] G. J. A. Hunter, R. C. L. Jenkins, and C. J. Tinsley, J. Phys. A **23**, 4547 (1990); R. G. Bowers, and B. Y. Yousif, Phys. Lett. A **96**, 49 (1983).
[21] Kun-Fa Tang and Jia-Zhen Hu, Acta Phys. Sin. **35**, 1048 (1986).
[22] S. G. A. Quadros and S. R. Salinas, Physica A **206**, 479 (1994).
[23] S. L. Schofield and R. G. Bowers, J. Phys. A **13**, 3697 (1980).
[24] T. Kaneyoshi and J. C. Chen, J. Magn. Magn. Mater. **98**, 201 (1991).
[25] A. F. Siqueira and I. P. Fittipaldi, J. Magn. Magn. Mater. **54**, 678 (1986).
[26] T. Kaneyoshi, Solid State Commun. **70**, 975 (1989).
[27] T. Kaneyoshi, J. Phys. Soc. Jpn. **56**, 2675 (1989).
[28] G. M. Buendia and M. A. Novotny, J. Phys.: Condens. Matter **9**, 5951 (1997).
[29] G. M. Buendia, M. A. Novotny, and J. Zhang, in *Computer Simulations in Condensed Matter Physics*, edited by D. P. Landau, K. K. Mon, and H. B. Schüttler (Springer, Berlin, 1994), Vol. VII, p. 223.
[30] Kun-Fa Tang, J. Phys. A **21**, L1097 (1988).
[31] G. M. Buendia and E. Machado, Phys. Rev. E **58**, 1260 (1998).
[32] Ce-Jun Liu, H. B. Schüttler, and Jia-Zhen Hu, Commun. Theor. Phys. **35**, 480 (2001).
[33] B. Y. Yousif and R. G. Bowers, J. Phys. A **17**, 3389 (1984).
[34] Kun-Fa Tang and Jia-Zhen Hu, J. Phys. A **20**, 3069 (1987).
[35] Yung-Li Wang, Felix Lee, and J. D. Kimel, Phys. Rev. B **36**, 8945 (1987).
[36] *Magnetic Molecular Materials*, Vol. 198 of *NATO Advanced Studies Institute, Series E: Applied Sciences*, edited by D. Gatteschi, O. Kahn, J. S. Miller, and F. Palacio (Kluwer Academic, Dordrecht, 1991).
[37] N. B. Wilding and P. Nielaba, Phys. Rev. E **53**, 926 (1996).
[38] K. Binder, J. Stat. Phys. **24**, 69 (1981).